mathematics

Article

Revisiting Long-Time Dynamics of Earth's Angular Rotation Depending on Quasiperiodic Solar Activity

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Abstract: Having taken into account the nonsymmetric form of Earth's surface (which is an oblate spheroid as the first approximation, with oblateness of approx. 1/300), we outline in the current research that additional large-scale torques stem from unbalanced (reactive) reradiating heat flows back into outer space. They arise during long-time dynamics of Earth's angular rotation depending on quasiperiodic solar activity. The key idea of our research supports the mainstream idea of most of the researchers in the scientific community regarding this matter. It stipulates that the activity of earthquakes strongly correlates with changes in the regime of Earth's spin dynamics during all periods of observation. We have demonstrated here that the long-time dynamics of Earth's angular rotation depends on the quasiperiodic solar activity by arising additional large-scale torques stemming from unbalanced (reactive) reradiating heat fluxes. The latter carry the momentum outside and at an unpredictable angle to the overall Earth's surface back into outer space (due to the nonsymmetric form of Earth's surface).

Keywords: Earth's angular rotation; activity of earthquakes; large-scale torques; reradiating heat fluxes; quasiperiodic solar activity

MSC: 70F15; 70F07

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Citation: Ershkov, S.; Leshchenko, D.; Prosviryakov, E. Revisiting Long-Time Dynamics of Earth's Angular Rotation Depending on Quasiperiodic Solar Activity. *Mathematics* **2023**, *11*, 2117. https://doi.org/10.3390/math11092117

Academic Editor: Panayiotis Vafeas

Received: 31 March 2023
Revised: 21 April 2023
Accepted: 27 April 2023
Published: 29 April 2023

1. Introduction: The Dependence of Earth's Angular Rotation Dynamics on Earth's Heat Flux Thermobalance via Quasiperiodic Solar Activity

This research aims to understand the long-time dynamics as well as the hidden causes for the sudden arising of strong earthquakes on the surface of Earth from the ancient times up to now. There exist profound fundamental studies (e.g., see [1–15]) which have been investigating the aforementioned problem over the period of ancient and modern science (including the recent publications related to the dynamics of the inner Earth's core [3,4]). Nevertheless, humankind can neither predict earthquakes nor avoid their catastrophic implications. The key idea in research [1–4] is that the activity of earthquakes strongly correlates with changes in the regime of Earth's spin dynamics during all periods of observation. Authors of [1] decomposed climatic time series into principal components and compared them with Earth rotation parameters. As a result, they have found quasiperiodic oscillations in the global mean Earth temperature anomaly. Similar cycles were also found in Earth's rotation variation. Moreover, Earth's angular rotation is correlated with the 60-year temperature anomaly during the last 160 years of observation. In [2], the analysis of Earth's rotation rate time series was performed using two different time series analysis methods. Both methods highlighted correlations between the detected anomalies in the

Earth's rotation rate time series and the world's earthquake occurrence with magnitude ≥ 7 and/or number of events ≥ 150 per day, within a time interval of ± 10 days between each earthquake. Authors of [3,4] argued, providing details and differences, that Earth's rotation has unpredictable internal dynamics (governed by Earth's inner core) correlated with the world's earthquake activity. Their main conclusion was that Earth's core is currently passing through a zero angular rotation state despite the persistence of differential rotation of Earth's inner core phenomenon. Changes over decades and its near halt were reported in [4].

We present here a physically reasonable ansatz with the aim to preliminary illuminate the influence of thermobalance of Earth's heat fluxes (stemming from quasiperiodic solar activity) on the dynamics of Earth's angular rotation. It is very important to first create an adequate physical model along with the aim to further develop the mathematical model for investigation of the problem under consideration.

As of now, most of the researchers use Liouville equations to model the viscous-elastic Earth's rotation (with one independent variable, time t):

$$\frac{\partial}{\partial t}\left[\vec{h}(t) + \vec{I}(t)\cdot\vec{\omega}(t)\right] + \vec{\omega}(t) \times \left[\vec{h}(t) + \vec{I}(t)\cdot\vec{\omega}(t)\right] = \vec{\tau}(t), \tag{1}$$

where $\vec{I}(t) \equiv \{I_1, I_2, I_3\}$ are the principal moments of inertia of Earth, and $\vec{\omega}(t) \equiv \{\omega_1, \omega_2, \omega_3\}$ are the components of the angular velocity vector along the proper principal axis; $\vec{h}(t) \equiv \{h_1, h_2, h_3\}$ is the part of angular momentum due to motion relative to the rotating reference frame; and $\vec{\tau}(t) \equiv \{\tau_1, \tau_2, \tau_3\}$ are the components of the net external torques, expressed via appropriate coordinates of the barycenter of Earth in a frame of reference fixed in the rotating body (in regard to the absolute system of coordinates X, Y, Z); signs $\cdot$ and $\times$ depict symbols of scalar (dot) and vector (cross) product of two chosen vectors here in (1).

This Liouville's system of Equation (1) generalizes the Euler equations of rigid-body rotation to the case of nonrigid (e.g., elastic or viscoelastic) Earth rotation dynamics where the net external torques are assumed to be zero [5]. A complete theoretical introduction to the problem of deriving vector formula (1), which demonstrates a viscoelastic Earth behavior, has been given in [5].

Nevertheless, both in Euler equations of rigid-body rotation and in Liouville's system of ODEs [5], physically reasonable and self-consistent external torques should be taken into account as governing external moments impacting the dynamics of the resulting solution of these equations.

The main idea of the current research is to apply the concept suggested in [16], explaining the chaotic motion of the center of the Sun with respect to the barycenter of the solar system. It is used hereby to explain the arising of additional torques, which should be taken into account in any theoretical model of Earth's rotation, Euler equations of rigid-body rotation, or Liouville's equations with nonzero external torques influencing Earth rotation dynamics. These additional torques arise since Earth absorbs energy from the Sun and then reradiates it into outer space as heat in a spatially unbalanced way. Indeed, we should take into consideration that there exists a disbalance to be calculated properly over the entire surface of Earth which is a nonsymmetric geoid [17].

We can see in Figure 1 that the geoid of Earth is supposed to be radiating light and energy fluxes back into outer space in an anisotropic way.

This is the reason why we should explicitly outline in the next section the thermobalance of the aforesaid Earth's heat fluxes where we could take into account (as the first approximation) the dynamics of the mean temperature of the Earth's surface with respect to time t only. Whereas in the given analysis, we will not take into consideration the essentially nonlinear process of the spatial distribution of temperature over the Earth's surface insofar as considering it in time. Such analysis will help us first to estimate the magnitude of this or those component(s) of Earth's heat flux thermobalance (comparing with, e.g., the average level of the tidal heating), and then to understand the nonlinear character of time-dependent Earth's heat fluxes reradiating into outer space from Earth's surface (not even spatially-dependent as the first approximation!). Thus, when further modeling the process of absorbing the energy from the Sun and then reradiating it into outer space, it is quite unrealistic to recognize the heat to be self-balanced (due to Earth being a nonsymmetric geoid). This is fundamental for understanding that additional large-scale torques, stemming from unbalanced (reactive) reradiating heat flows back into outer space, should arise during long-time dynamics of Earth's angular rotation depending on quasiperiodic solar activity.

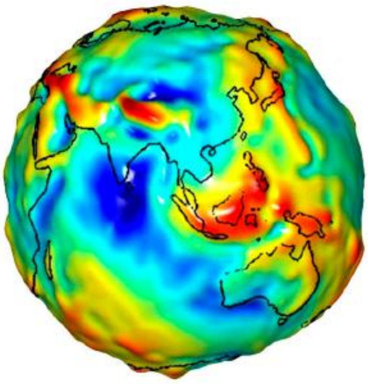

Figure 1. Schematically illustrating the nonsymmertic geoid of Earth, which is a spheroid as the first approximation (non-sphericity circa equals to 1/300 [17]). Color is changing from blue (downlands or basins) to red (uplands) according to arrangement with respect to surface of Earth (according to the level of World Ocean).

2. Methodology

2.1. Estimation of Thermobalance of Earth's Heat Fluxes via Quasiperiodic SolarActivity

In this section, we will obtain a nonlinear but ordinary differential equation for the mean temperature of Earth's surface dynamics, which could then be solved by analytical or numerical methods. Whereby, solving the equation for Earth's surface temperature dynamics is reduced to a *Riccati*-type ODE of 1st order regarding the time t. With this prerequisite, an elegant quasiperiodic solution was obtained for the aforementioned equation, describing Earth's surface temperature dynamics over a long-time period.

Let us consider Earth to be a black body [18] of spherical shape (as the first approximation) with radius R which absorbs and then partially reradiates the incoming heat fluxes, assuming that Earth's surface should have the mean temperature $T(t)$ at the moment of time t during this process. To estimate Earth's heat flux thermobalance, we should take into account the net amount of all the heat fluxes, namely:

- (1) The solar radiation energy flux [19,20]:

$$F_{ext} = f \cdot (\pi R^2) \cdot (1 - A), \tag{2}$$

where f is the measure of flux density of the mean solar electromagnetic radiation (or the solar irradiance) per unit of area which would be measured on a plane perpendicular to the rays, at a distance of one astronomical unit (AU) from the Sun, f = 1.361 kilowatts per square meter (kW/m^2) at solar minimum [19]; R is the radius of Earth; A is Earth's spherical *albedo*, A = 0.29 [20];

- (2) Surface of Earth reradiates all the energy fluxes to outer space according to the Stefan-Boltzmann law [21]:

$$F_{out\ (1)} = 4\, \pi\, R^2 \cdot (\sigma \cdot T^4) \cdot B, \tag{3}$$

where σ is the constant of Stefan-Boltzmann law [21], B is the coefficient depending on the opaqueness of the atmosphere for the infrared wavelengths (coefficient B is known to be varying depending on the density of water vapor in clouds [19,20]);

- (3) The additional losses for evaporation of water from the surface of the ocean [20] should also be taken into account:

$$F_{out\ (2)} = 4\,\pi\,R^2 \cdot (h \cdot \rho_w \cdot Q), \tag{4}$$

where $Q = Q_0 - \eta{\cdot}T = (25 - 0.024{\cdot}T){\cdot}10^5$ [J/kg] is the total amount of heat of water evaporation from the unit of the ocean's square per unit of time (dimension of constant 0.024 is in [K^{-1}] since T is the absolute Kelvin temperature, compatible with the Stefan-Boltzmann law (2)); h is the average height of water in mm, which is assumed to be evaporating from the surface of the ocean per unit of time, $h \approx 1000$ mm/year $=3 \times 10^{-8}$ m/s; $\rho_w \approx 1024$ kg/m^3 is the density of seawater (if the time scale is annual or decadal, it will be fine to assume that the evaporation is equal to condensation/precipitation; here condensation is the opposite of the evaporation process);

- (4) We should additionally take into account the losses for the convection according to the NASA Earth Observatory report [20] as given below

$$F_{out(3)} = \alpha \cdot F_{ext}, \tag{5}$$

where α is the appropriate dimensionless coefficient of proportionality, α = 0.05 (5%).

Let us note that, according to the data in the NASA Earth Observatory report [20], only 48% of the initial solar radiation energy flux F_{ext} reaches the Earth's surface (we denote the appropriate coefficient as β = 0.48). It means that $B = \beta - \alpha - (F_{out\ (2)}/F_{ext})$, where the proper ratio $(F_{out\ (2)}/F_{ext}) = 0.25$ is considered at the temperature approx. 17 °C ≈ 290.2 K; but the losses during the reradiation, according to the NASA Earth Observatory report [20], are approx. 18% of the initial solar radiation energy flux F_{ext}, $F_{out\ (1)} = 0.18\,F_{ext}$.

Thus, we can estimate Earth heat flux thermobalance near the surface of Earth, without accounting for the effects of heat transfer by turbulence or diffusion inside the ocean:

$$\frac{d\,H}{d\,t} = \beta \cdot F_{ext} + F_{int} - \left(F_{out\ (1)} + F_{out\ (2)} + F_{out\ (3)}\right) \tag{6}$$

where H is the effective enthalpy of Earth's surface; F_{int} is the source of internal heat generation (of unknown nature which should be determined).

So, we should obtain from Equations (2)–(6) as below:

$$\frac{d\,(C \cdot T)}{d\,t} = \frac{(\beta - \alpha) \cdot F_{ext} + F_{int}}{4\,\pi\,R^2} - \sigma \cdot T^4 \cdot B - h \cdot \rho_w \cdot (Q_0 - \eta \cdot T) \tag{7}$$

where t is the time; C is the isobaric heat capacity of the Earth's surface per unit of area. If we designate:

$$\Delta f_{int} = \frac{F_{int}}{4\,\pi\,R^2}$$

then Equation (7) could be transformed as

$$\frac{d\,(C \cdot T)}{d\,t} = \left\{f \cdot \frac{(\beta - \alpha) \cdot (1 - A)}{4} + \Delta f_{int}\right\} - \sigma \cdot T^4 \cdot B - h \cdot \rho_w \cdot (Q_0 - \eta \cdot T) \tag{8}$$

where generally $\Delta f_{int} = \Delta f_{int}\,(t)$, while, we should specifically note that $f = f\,(t)$ or is generally time-dependent due to variations of the mean solar electromagnetic radiation (or the solar irradiance) flux density per unit of the area due to the well-known effect of *Milankovitch cycles* [22].

As we can see, Equation (8) is the generalization of the *Riccati*- or *Abel*-type equations [23]. Namely, *Riccati*-type Equation (8) is an ordinary differential equation of the 1st order, whose nonlinearity evidently comes from the Stefan-Boltzmann law. Without the additive term of $\Delta f_{int} = \Delta f_{int}\,(t)$, the derivation of analytical solutions would be available, using the standard method of variables separation. The aim of our research is not an in-depth analysis of solutions' existence or properties of (8) (including the numerical simulations and the influence of initial condition), but rather a qualitative analysis based on finding the *thermodynamic equilibrium* {under condition $d\,T = 0$ in (8)}, which is available in the next section. It is required to estimate Earth's heat flux thermobalance and is important to demonstrate the existence of additional large-scale torques stemming from unbalanced (reactive) reradiating heat flows back into outer space. Nevertheless, it is noted that due to a very special character of solutions for ordinary differential equations of *Riccati's* type, their general solutions are known to have appropriate jumping of solutions' components at some definite moments of time t_0 (depending on the initial conditions) [24].

Thus, there exists a possibility of the scenario for sudden *global change* in mean temperature dynamics on Earth's surface, global climate change on Earth, as well as in Earth's biosphere's environment (let us recall the well-known *Little Ice Age* at the Maunder's minimum of the solar activity [25]).

2.2. Quasiperiodic Dynamics of Earth Surface Mean Temperature

According to the modern climatology data, the average temperature near the surface of the world ocean [26] is approx. 17 °C ≈ 290.2 K.

If we assume $\{\Delta f_{int}, f\} = const$ in (8), Equation (8) could be solved analytically as below (let us additionally note that for the solid surface of Earth, the heat capacity is not less than twice lower than heat capacity at the ocean's surface, but density is more than twice as high, accordingly):

$$\int \frac{dT}{\{f \cdot \frac{(\beta-\alpha)\cdot(1-A)}{4} + \Delta f_{int}\} - \sigma \cdot T^4 \cdot B - h \cdot \rho_w \cdot (Q_0 - \eta \cdot T)} = \int \left(\frac{1}{C}\right) dt \quad (9)$$

where the left side of Equation (9) could be transformed into the proper *quasiperiodic analytical* expression [27] in regards to the function $T(t)$. In mean-time scale, present temperature $T(t)$ could be represented as a set of *quasiperiodic* cycles due to approx. constant coefficients in the aforementioned *Riccati*-type equation:

$$a = \left(\{0.25 f \cdot (\beta - \alpha) \cdot (1 - A) + \Delta f_{int} - h \cdot \rho_w \cdot Q\} / \{\sigma \cdot B\}\right)^{\frac{1}{4}} \quad (10)$$

If we exclude the influence of any source of internal heat generation Δf_{int} in (9), the maximum for the temperature of the ocean's surface could be calculated for the case of *thermodynamic equilibrium* {it means $d\,T = 0$ in (8)} from the expression (10) as shown below:

$$a_0 = \left(\{0.25 f \cdot (\beta - \alpha) \cdot (1 - A) - h \cdot \rho_w \cdot Q\} / \{\sigma \cdot B\}\right)^{\frac{1}{4}} = 255.4\ \text{K} \cong -17.8\ ^\circ\text{C}.$$

The aforementioned parameters (in the expression above) are taken as follows: $f = 1.361$ (kW/m^2), $\beta = 0.48$, $\alpha = 0.05$, $A = 0.29$; $Q = 18.9 \times 10^5$ [J/kg], $h \approx 3 \times 10^{-8}$ m/s, $\rho_w = 1024$ kg/m^3; $\sigma = 5.67 \times 10^{-8}$ [W/(m^2·K^4)], $B = 0.19$.

Then we obtain from (2)–(5):

$$\begin{gathered} F_{out\ (1)} = 0.19\, F_{ext},\ F_{out\ (2)} = 0.24\, F_{ext},\ F_{out\ (3)} = 0.05\, F_{ext} \\ \rightarrow\ F_{out\ (1)} + F_{out\ (2)} + F_{out\ (3)} = 0.48\, F_{ext} \end{gathered} \quad (11)$$

Thus, we should conclude that the process of heating by using solar activity would only yield an average value for Earth's surface temperature of approx. eighteen degrees below zero (Celsius scale). Therefore, $\Delta f_{int} \neq 0$ in (9). Moreover, the essential part of the sum of sources for internal heat generation Δf_{int} is, of course, the *greenhouse effect* [20].

NASA Earth Observatory reported in [20] that 0.48 F_{ext} is the total sum of heat fluxes first to be absorbed by Earth's surface (from the incoming flux of solar radiation F_{ext}). Then such total flux should be reradiating from Earth's surface to the Earth's atmosphere.

Moreover, according to the NASA Earth Observatory report [20], the aforementioned heat fluxes (from Earth's surface to the Earth's atmosphere) should be calculated as below:

$$\begin{gathered} F_{out\ (1)} = 0.18\ F_{ext},\ F_{out\ (2)} = 0.25\ F_{ext},\ F_{out\ (3)} = 0.05\ F_{ext} \\ \rightarrow\ \ F_{out\ (1)} + F_{out\ (2)} + F_{out\ (3)} = 0.48\ F_{ext} \end{gathered} \tag{12}$$

this result approximately corresponds to the net sum of heat fluxes (11) above.

To achieve the observed temperature of 290.2 K [26] on Earth's surface as a result of our calculations, we should take into account the appropriate parameters in (10) as follows: f = 1.361 (kW/m^2), β = 0.48, α = 0.05, A = 0.29; $\Delta f_{int} = 0.134 \cdot F_{ext}$, $Q = 18 \times 10^5$ [J/kg], $h \approx 3 \times 10^{-8}$ m/s, ρ_w = 1024 kg/m^3; $\sigma = 5.67 \times 10^{-8}$ [W/(m^2·K^4)], B = 0.201.

In this case, Equations (2)–(5) would yield:

$$\begin{gathered} (F_{out\ (1)} + \Delta f_{int}) = (0.335 - 0.134)F_{ext} \cong 0.201\ F_{ext},\ F_{out\ (2)} = 0.23\ F_{ext},\ F_{out\ (3)} = 0.05\ F_{ext} \\ \rightarrow\ \ F_{out\ (1)} + F_{out\ (2)} + F_{out\ (3)} \cong 0.48\ F_{ext} \end{gathered} \tag{13}$$

where the result 0.48 F_{ext}, calculated in (13), corresponds to the net of heat fluxes (12) above: indeed, $F_{out\ (1)}$ in (13) increased relative to the magnitude in (12) by 11% (equals to 0.02 F_{ext}), and $F_{out\ (2)}$ decreased by 9% (equals to the same amount of 0.02 F_{ext}). Using the total amount of heat fluxes in (13), we obtain the proper magnitude of 0.48 F_{ext}, as we can see from Equations (11) and (12) above.

Let us especially note that, according to the data in NASA Earth Observatory report [20], the amount of *greenhouse effect* should be $\Delta f_{int} = \chi \cdot F_{ext}$, where χ = 0.134 or 13.4% of the initial solar radiation energy flux F_{ext}.

As to the estimation of the solar radiation energy fluxes, it is an easy matter (see (2), where the radius of the Earth, R = 6,371,000 m as a first approximation): after the calculations, we obtain the total amount of $F_{ext.} \cong 0.48 \times 111{,}072$ Terawatts $\cong$ 53,315 (TW). Thus, according to the data in [20], we should obtain the amount of ~7144 (TW) for the 13.4% of the *greenhouse effect*.

Finally, the current dynamics of Earth's surface temperature (8) suggests a quasi-periodicity: indeed, if we recall the *Milankovitch* cycles [22] for all the key parameters in (8) (even for $f = f(t)$, in the general case), it should obviously mean the *quasiperiodic* character of the solutions of Equation (8).

3. Results and Analysis

As we can see, the aforementioned calculations demonstrate that the process of Earth's heating by only solar activity is not sufficient to obtain the observed mean value of the temperature on Earth's surface. Indeed, simple absorbing of the incoming solar radiation is apparently not sufficient for Earth's heating, as the mean surface temperature is much higher than the black body temperature corresponding to the energy flux of the incoming solar radiation.

This means that other sources of heating should exist in the case of Earth, i.e., the *inner* sources of global heating for Earth itself as a planet. The most obvious source of internal heat generation (within the system "the Earth + its atmosphere") is, of course, the *greenhouse effect*. The main reason is that almost all the radiation emitted from Earth's surface is absorbed by the atmosphere, and then reradiated back to Earth. We should also mention the *tidal* heat generation (approx. 3 TW for Earth [19,28]) as the *inner* source of internal heat generation (e.g., the additional tidal heating via interacting of fluid tides or Rossby waves with the solid structure of the borders of coastline in the ocean). As we can see, all the considerable additional inner sources of internal heat generation (namely, the *tidal* heating in amount approx. 3–4 TW) are much less than the estimation for the *greenhouse effect* in the previous section (~7144 TW).

Considering the nonsymmetric form of Earth's geoid (which is an oblate spheroid as the first approximation, with a coefficient of non-sphericity equaling approx. 1/300 [17]), it becomes obvious that additional large-scale torques, stemming from unbalanced (reactive) reradiating heat flows back into outer space, should arise during long-time Earth angular rotation dynamics depending on quasiperiodic solar activity.

The main analytical result of the current research should be specifically outlined: due to *Milankovitch* cycles [22] (which should be taken into account for all the variable key parameters in (8)), the aforementioned solution of the Equation (8) for the dynamics of the mean temperature of the Earth's surface should have the quasiperiodic character of *Riccati*-type during a long period of time t. Thus, we could present temperature $T(t)$ as a set of quasiperiodic cycles (Figures 2 and 3).

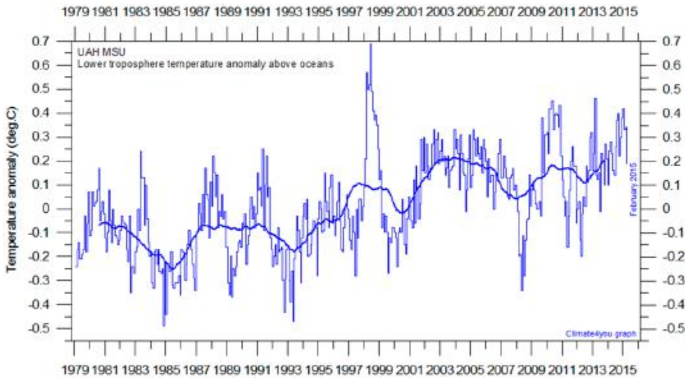

Figure 2. The global monthly (average) temperature of the lower troposphere above oceans since 1979, according to the data of the University of Alabama at Huntsville, USA. The thin line indicates 3-month values, and the thick line represents the 39-month average.

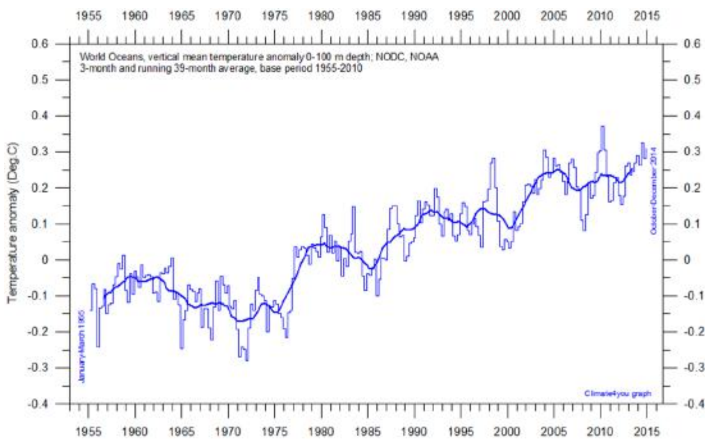

Figure 3. World oceans vertical (average) temperature 0–100 m depth since 1955; the thin line indicates 3-month values and the thick line represents a 39-month average.

Last, but not least, it is interesting to push slightly forward for the simple energy balance models to include the main processes at the interface between surface and atmosphere (by simplifying assumptions, e.g., the ice-albedo feedback mechanism having been excluded from the consideration), along with a time-dependent dynamics representing the small and subtle energy imbalances leading to climate variability. The last assumption stems from the *Riccati*-type character of the solutions of Equation (8). Indeed, we should note that due to the special character of the solutions of *Riccati*-type ODEs, there is the possibility for a sudden *jump* in the magnitude of the solution at some time t_0 [23].

Let us also briefly analyze the conclusions provided in the studies [6–10,12–15] made by other researchers. In [6], the author applied the theory of canonical perturbations to study Earth's rotation and, together with his coauthor, developed the approach suggested earlier in their work [7] applicable to attitude dynamics studies of Earth rotation via osculating and non-osculating Andoyer variables (they also introduced in their study the Delaunay variables that chosen to be the orbital elements). The author of [8] discussed sufficient evidence (according to his opinion) for correlating changes in Earth's rotation with the changes in atmospheric angular momentum and the strongest earthquakes. In the classical work [9], the author introduced his theory of the rigid Earth rotation. Together with his coauthor, he developed the approach [9] suggested earlier in their work [10] devoted to the theory of nutation for the rigid-Earth model with improvements due to an extension of the theory of the second order (considering in their study triaxiality of Earth and other effects of the second order). In fundamental research [12], the author presented his theoretical approach with respect to the dynamics of rotation of solid bodies in the solar system. Specifically, the effects of elastic distortion, non-principal axis rotation, precessing orbits, and internal dissipation on the rotation of a solid solar system bodies were analyzed. Examples of applications include spin-orbit coupling, generalized Cassini laws, tidal evolution, etc. In [13] the theory of the rotation of Earth around its center of mass was developed as a similar representation of disturbed orbital motion obtained in the planetary theory by Lagrange's method of elements variation. The author of [14] suggested a solution to the rotation of the elastic Earth by the method of rigid dynamics—namely, Hamiltonian formulations for rotation of a deformable body (elastic Earth) and the derivation of the equations of motion from it. In the comprehensive research [15], the authors study the correlations between the variability of regimes of Earth's rotation rate and cyclic processes in geodynamics within Earth's body, including seismic activity. Their main conclusion is that the rotation rate of a planet determines its uniaxial compression along the axis of rotation and external geometry (and topology) of Earth's surface. This is based on their conclusion that the Earth's ellipticity variations (see Figure 1 above) are caused by angular rotation rate variations.

4. Conclusions

The proper estimation of the average Earth's surface temperature depending on heating by the solar radiation energy flow is investigated here.

A mathematical model of Earth heat flux thermobalance is suggested for such an estimation. Using well-known data (i.e., albedo of Earth's surface, constant of the heat radiation from the sun per square meter of the surface of Earth's outer atmosphere, etc.), it can be concluded that the process of heating by only solar activity is not sufficient to obtain the real temperature of Earth's surface. Estimation yields the average temperature approx. –18 degrees below zero (Celcius scale) in this case. It means that there exist other sources of heating for Earth; we suppose such sources should be the *inner* sources of heating for Earth itself as a planet (*greenhouse effect*).

The main analytical result of the current research suggests that the solution of the basic equation describing Earth's surface temperature dynamics should have the *quasiperiodic* character of *Riccati*-type during a long period of time t (due to *Milankovitch* cycles). In the mean-time scale, temperature $T(t)$ could be presented as a set of *quasiperiodic* cycles due to approx. constant coefficients in the aforementioned equation of *Riccati*-type.

While according to our estimation, *tidal* heating gives the estimation in the amount of ~3–4 TW, we should mention other possible sources of heating of Earth's surface [29–32]. The main source of internal heat generation within the system "Earth + Earth's atmosphere" is, of course, the *greenhouse effect* (~7144 TW). Much of the radiation emitted from the surface of Earth is absorbed by the atmosphere (due to the heating of the atmosphere by greenhouse gases) and is then reradiated back to Earth for additional heating. In conclusion, some elements of the nonpassive role of the atmosphere, also related to the *greenhouse effect*, are already integrated in the description even when the relative forcing is not explicitly present.

Taking into account the nonsymmetric form of Earth's surface (which is an oblate spheroid as the first approximation, with a coefficient of non-sphericity equaling approx. 1/300 [17]), we should specifically outline and conclude that additional large-scale torques stemming from unbalanced (reactive) reradiating heat flows back into outer space (~7144 TW) arise during long-time dynamics of Earth's angular rotation depending on quasiperiodic solar activity. The key idea in research [1–4,30] is that the activity of earthquakes strongly correlates with the changes in the regime of Earth's spin dynamics during all periods of observation. We have demonstrated in our research that the long-time dynamics of Earth's angular rotation depends on quasiperiodic solar activity via arising of additional large-scale torques stemming from unbalanced (reactive) reradiating heat fluxes. The latter carry momentum outside and at unpredictable angle to the overall Earth's surface back into outer space (due to the nonsymmetric form of Earth's surface).

The given research does not consider the dependence of Earth's angular rotation state on Earth's atmosphere via arising of additional large-scale decelerating torques that stem from mechanical resistance or the influence of turbulent flows in the atmosphere on Earth's surface. This matter has been described in detail in [31,32]. The heating outcome from such large-scale decelerating torques can be compared by its order with the effect of the aforementioned *tidal* heating (3–4 TW) versus the *greenhouse effect* (~7144 TW). Last, but not least, it is worth noting that the case when additional large-scale torques, stemming from (reactive) reradiating heat flow from Earth's surface back into outer space, are assumed to be a *zero* net value integrated over the entire surface of Earth obviously corresponds to the case of the ideally symmetric form of Earth's surface close to a sphere. We have outlined in our research the significance of accounting for additional large-scale torques stemming from unbalanced (reactive) reradiating heat flows back into outer space. They arise during long-time dynamics of Earth's angular rotation depending on quasiperiodic solar activity due to the asymmetric form of Earth's surface. All these statements have been included in the title, the word "Revisiting" means that, as of now, according to the best of our knowledge, no research includes or accounts for such phenomenon in dynamical model of Earth's rotation (even in various types of torques used in formulation of Liouville Equation (1) of Earth's rotation dynamics).

Details regarding semi-analytical formulation of how Milankovitch cycles can be approximated during the time period of chosen numerical scheme for the calculation can be found in [33]. As far as we know, the scale of Milankovitch cycles refers to thousands of years; nevertheless, they should be considered in the mathematical model of Earth heat flux thermobalance. They should be taken into account at formulation of the dynamical model of Earth's rotation for such model to be physically reasonable and self-consistent according to the data of long-time astrometric observations.

Let us note that the phenomenon of differential rotation, tackled in [4], was also considered in detail in [34] for celestial mechanics applications. Solutions for ordinary differential equations of *Riccati's* or *Abel's* types were considered in [35] with respect to the unregular regime of celestial small body (satellite of planet) rotation.

Author Contributions: Conceptualization, S.E.; methodology, S.E.; software, E.P.; validation, S.E., D.L. and E.P.; formal analysis, S.E.; investigation, S.E.; data curation, D.L.; writing—original draft preparation, S.E.; writing—review and editing, S.E.; visualization, S.E.; supervision, D.L.; project administration, E.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data are contained within the article.

Conflicts of Interest: Authors declare that there is no conflict of interests regarding publication of article.

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